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DEPARTMENT OF ENGINEERING MECHANICS

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Commanding Officer
Office of Naval Research
Chicago Branch
The John Crerar Library Bldg.
86 East Randolph Street
Chicago 1, Illinois

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Progress Report No. 1

Dear Sir:

This progress report covers the preliminary stages of the work on "Forces Experienced by Submerged Harmonically Oscillating Flat Plates". The report begins with a mathomatical discription of the fluid motion around moving oscillating plates. This description is then formulated by a transformation into two parts. The first part is the steady motion of the fluid around a flat plate moving with a constant velocity. The second part corresponds to the disturbed motion of the fluid arising from the oscillation of the plate. The solution to the first part is well known; it was given by Kotchin in 1937. The second part is much more difficult, and it is with this phase that we are actively engaged.

Very Sincerely,

Adolf G. Strandhagen

Adolf G. Strandhagen
Director of Project

Distribution: ONR Chicago (1)

ONR Washington, Code 438 (5)

The equation of fluid motion around a submerged body moving in the direction of positive x axis with a constant velocity U is

$$\nabla^2 \phi(x,y,t) = 0 \tag{1}$$

where ϕ is a function of time and point coordinates in the moving frame.

The free-surface condition can be shown to be:

$$\frac{\partial^2\phi}{\partial t^2} - 2U\frac{\partial^2\phi}{\partial t\,\partial x} + U^2\frac{\partial^2\phi}{\partial x^2} + g\frac{\partial\phi}{\partial y} = 0\,, \qquad y = 0 \tag{2}$$

On the surface of the lifting surface we have the flow conditions:

$$\frac{\partial\phi}{\partial y} = v_o(x) + v(x)\,\exp(ikt), \qquad (y = -f, \quad |x| \leq b) \tag{3}$$

where $v_o(x) = -U\theta$ is the normal component of the velocity of the thin flat plate in a translaroty motion; and θ is the mean angle of attack. The normal velocity corresponding to the oscillation of the wing with a frequency k is represented by $v(x)\,\exp(ikt)$ where $v(x)$ is the normal component of the amplitude due to rotational oscillations. The mean position of the plate is taken to by at $y = -f$, i.e. at distance of $|f|$ below the undisturbed position of the free surface. The chord of the plate is 2b.

The pressure inside the fluid is given by the expression:

$$p - p_o = -\rho_o\left(\frac{\partial\phi}{\partial t} + U\frac{\partial\phi}{\partial x}\right) - \rho_o g z,$$

where p_o is the atmospheric pressure, and ρ_o is the density at infinity.

As the pressure change $p - p_o$ must be continuous except when crossing the plate, it follows that a further condition may be taken in the form:

$$p - p_o = 0, \text{ when } y = -f, \quad |x| \geq b \tag{4}$$

A finite velocity at the trailing edge implies an additional condition:

$$\frac{\partial \phi}{\partial x} \text{ is finite, when } y = -f, \text{ and } x = -b \tag{5}$$

Now assuming the perturbed motion has reached a steady state, we transform Equations (1) through (5) by letting

$$\phi(x,y,t) = \phi_0(x,y) + \phi_1(x,y)\exp(ikt) \tag{6}$$

where $\phi_0(x,y)$ is the velocity potential corresponding to a steady motion of the fluid for a plate moving with a constant velocity U, and $\phi_1(x,y)\exp(ikt)$ represents the velocity potential of the disturbed oscillatory motion of the fluid. The result of substituting (6) into the above equations yields:

$$\left.\begin{aligned}
&\frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial^2 \phi_0}{\partial y^2} = 0 \\
&\frac{\partial \phi_0}{\partial y} = v_0(x), \text{ when } y = -f, \quad |x| \leq b \\
&\frac{\partial \phi_0}{\partial x} \quad \text{is finite, when } y = -f, \text{ and } x = -b \\
&U\frac{\partial \phi_0}{\partial x} + gz = 0, \text{ when } y = -f, \quad |x| \geq b \\
&\frac{\partial^2 \phi_0}{\partial x^2} + \frac{g}{U^2}\frac{\partial \phi_0}{\partial y} = 0, \text{ when } y = 0
\end{aligned}\right\} \tag{7}$$

and for perturbed motion we obtain:

$$\left.\begin{aligned}
&\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} = 0,\\
&\frac{\partial \phi_1}{\partial y} = v(x), \text{ when } y = -f, \quad x \leq b\\
&\frac{\partial \phi_1}{\partial x} \text{ is finite when } y = -f, \text{ and } x = -b\\
&i k \phi_1 + U \frac{\partial \phi_1}{\partial x} = 0, \text{ when } y = -f, \text{ and } |x| \geq b\\
&\frac{\partial^2 \phi_1}{\partial x^2} - 2i\left(\frac{k}{U}\right)\frac{\partial \phi_1}{\partial x} + \frac{g}{U^2}\frac{\partial \phi_1}{\partial y} - \left(\frac{k}{U}\right)^2 \phi_1 = 0, \text{ when } y = 0
\end{aligned}\right\} \quad (8)$$

The solution for ϕ_0, as mentioned above, corresponds to the steady motion of a fluid around a flat plate which is moving below the free surface at constant velocity. This solution is known; Kotchin in 1937 published a solution for this case. Therefore, we shall proceed to a discussion of the solution for ϕ_1.

It is evident that the conditions (8) imply not only the usual type of boundary conditions but also conditions to be satisfied in the interior of the fluid. The solution to conditions (8) is proceeding along three directions: (1) solution of a single pulsating source moving with a constant velocity below the free surface. This solution will be adopted to the case of distribution of sources along the countour of the oscillatory plate. (2) Transformation of coordinates with a view of reducing conditions (8) to well-known solutions such as an oscillating flat-plate placed below the free surface but with zero speed of advance. This method has not been completely analyzed for all details, but offers some possibilities, and (3) a head-on approach to satisfy mathematically all the conditions (8); this direction has not been seriously pursued to date.

The study of single oscillating source mentioned-above shows that there exist three ranges of k for which the propagation of disturbances takes on distinct characters:

(a) $k < \frac{1}{4}(g/U)$; two harmonic wave trains of different wave length and speed propagate to infinity on the downstream side of moving source in addition to the local disturbances which travel with source.

(b) $g/4U < k < g/2U$; two harmonic wave trains propagate in opposite directions with respect to the moving source, i.e.

one wave propagates downstream to infinity, while the other wave propagates upstream to infinity, in addition to the local disturbances traveling with the source.

(c) k $g/2U$; In this case no disturbances are propagated in either direction, and only local disturbances will be produced which travel with the source.

Figure illustrating the coordinates employed in the above discussion of conditions (7) and (8).